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Visibility of a Lunar Satellite from an Earth Ground Station

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Introduction

ANALYSIS is presented for the solution of the visibility problem of a satellite of the moon from a ground station located on earth. The first portion of the line-of-sight communication problem is solved by obtaining a single transcendental equation in the eccentric anomaly of the lunicentered satellite.

Future space missions, specifically the conquest of the moon, as now envisioned under the Apollo project, present special communication problems. This paper investigates the optical or radar visibility of a satellite in an orbit about the moon from a specified station on the earth. The analysis is investigated from an astrodynamic point of view.

As will be seen, this problem becomes rather complex and should be separated into two different but related conditions that insure visibility of the lunar satellite. The first of these conditions, called herein the critical viewing time condition, i.e., when the satellite is not obscured by the bulge of the earth, is the problem attacked in this note. Condition two, the lunar eclipse situation, i.e., when the satellite is hidden by the moon, can be treated by a slight modification of eclipse theory.[4] Both conditions must be satisfied in order to insure a direct optical contact between the specified terrestrial station and the orbiting lunar satellite.

Critical Viewing Time Condition

Consider a ground station located in a geocentric, inertial coordinate system[1,2] illustrated in Fig. 1, with coordinates ϕ' and θ defined by $\phi' \equiv$ station geocentric latitude and $\theta \equiv$ station local sidereal time. The rectangular coordinates of the station coordinate radius vector **R** may be shown to be

$$X = -G_1 \cos\phi \cos\theta \qquad Y = -G_1 \cos\phi \sin\theta$$
$$Z = -G_2 \sin\phi \tag{1}$$

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where, as developed in Ref. 1,

$$G_1 \equiv \frac{a_e}{[1 - (2f - f^2)\sin^2\phi]^{1/2}} + H$$

$$G_2 \equiv \frac{(1 - f)^2 a_e}{[1 - (2f - f^2)\sin^2\phi]^{1/2}} + H$$

with the geodetic latitude ϕ defined by

$$\phi = \tan^{-1}[\tan\phi'/(1 - f)^2] \qquad -(\pi/2) \le \phi \le (\pi/2)$$

and a_e = equatorial radius of earth, f = flattening of adopted ellipsoid, and H = station elevation above and measured normal to the surface of the adopted ellipsoid.

Consider the position of the moon defined in the same coordinate frame by vector $\mathbf{r}_m$, i.e., the position vector of the moon's center with respect to the geocenter. The coordinates of $\mathbf{r}_m$ can be obtained from the *American Ephemeris and Nautical Almanac*.[5] From the selenocenter, the position vector of a lunar satellite, $\mathbf{r}_s$, can be introduced and the vector $\mathbf{r}_c$ defined by

$$\mathbf{r}_c = \mathbf{R} + \mathbf{r}_m + \mathbf{r}_s \tag{2}$$

Figure 2 affords further clarification of the vector construction.

As can be seen from an observing station located at point A, it is always possible to observe satellite S, if S is itself not eclipsed by the moon. However, as the station rotates into position A' because of the sidereal rotation of the earth, a critical situation will be reached. This critical situation is of course caused by the line-of-sight interference of the earth, which has moved into a position between the lunar satellite and the observing station. For a spherical earth, the critical angle occurring at A' caused by line-of-sight interference is $\pi/2$, but for the sake of generality, and since the observing station may have definite observational constraints because of mountains, etc., let the critical angle be $\pi/2 + h$, where h is the minimum acceptable elevation angle of the observing station. For an oblate planet, h must be augmented by the angle ζ where

$$\zeta = \cos^{-1}\left[\frac{(G_1 \cos^2\phi + G_2 \sin^2\phi)}{R}\right] \qquad 0 \le \zeta \le \frac{\pi}{2}$$

In light of the previous discussion, the critical situation occurs when

$$\mathbf{r}_c \cdot \mathbf{R} = r_c R \cos(\pi/2 + h) = -r_c R \sinh \tag{3}$$

Substituting for $\mathbf{r}_c$ from Eq. (2) results in the relationship

$$(\mathbf{R} + \mathbf{r}_m + \mathbf{r}_s)\cdot\mathbf{R} = -r_c R \sinh \tag{4}$$

or

$$(\mathbf{r}_m + \mathbf{r}_s)\cdot\mathbf{R} = -R^2 - r_c R \sinh \tag{5}$$

Let the two unit vectors $\mathbf{P}_s$, $\mathbf{Q}_s$ be introduced where $\mathbf{P}_s$ is a vector pointing toward perigee of the lunicentered orbit, and $\mathbf{Q}_s$ is advanced to $\mathbf{P}_s$ by a right angle in the plane and direction of motion. Furthermore, if the lunicentered orbit

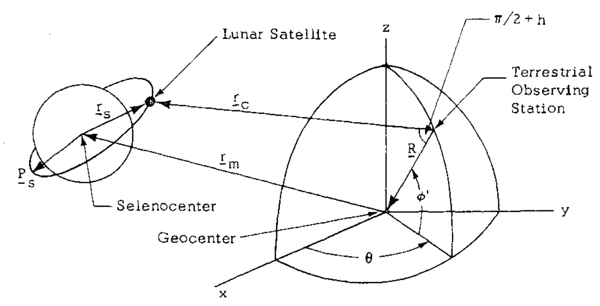

Fig. 1 Orbit geometry.

is defined by the elements a_s, e_s, i_s, Ω_s, ω_s, and T_s, where a_s is the semimajor axis of the lunicentered vehicle, e_s is the eccentricity of the lunicentered vehicle, i_s is the orbital inclination of the lunicentered vehicle with respect to the earth's equatorial plane, Ω_s is the longitude of the ascending node referred to the intersection of the planes of motion of the vehicle and the earth's equator, ω_s is the argument of perigee referred to the intersection of the planes of motion of the vehicle and the earth's equator, and T_s is the time of perifocal passage of the lunicentered vehicle, then $\mathbf{P}_s$ and $\mathbf{Q}_s$ are given by

$$\left.\begin{aligned} P_{xs} &= \cos\omega_s \cos\Omega_s - \sin\omega_s \sin\Omega_s \cos i_s \\ P_{ys} &= \cos\omega_s \sin\Omega_s + \sin\omega_s \cos\Omega_s \cos i_s \\ P_{zs} &= \sin\omega_s \sin i_s \\ Q_{xs} &= -\sin\omega_s \cos\Omega_s - \cos\omega_s \sin\Omega_s \cos i_s \\ Q_{ys} &= -\sin\omega_s \sin\Omega_s + \cos\omega_s \cos\Omega_s \cos i_s \\ Q_{zs} &= \cos\omega_s \sin i_s \end{aligned}\right\} \quad (6)$$

It should be noted that i, ω, Ω could be referred to the equatorial plane of the moon and then rotated at the lunicenter to yield i_s, ω_s, and Ω_s, i.e., the orientation angles of the lunicentered orbit referred to the fundamental plane of the earth.

The $\mathbf{P}_s$ and $\mathbf{Q}_s$ unit vectors permit the following relationship to be written[1,2]:

$$\mathbf{r}_s = x_{\omega s}\mathbf{P}_s + y_{\omega s}\mathbf{Q}_s \quad (7)$$

where

$$x_{\omega s} = a_s(\cos E - e_s) \qquad y_{\omega s} = a_s\,(1 - e_s^2)^{1/2} \sin E$$

with E = the eccentric anomaly of the lunicentered vehicle. Equation (7) is a mapping from two space to three space and expresses the rectangular coordinates of the lunicentered vehicle in the geocentric coordinate frame illustrated in Fig. 1. By consequence of this mapping, Eq. (5) can be written as

$$\{\mathbf{r}_m + a_s(\cos E - e_s)\mathbf{P}_s + a_s\,[(1 - e_s^2)^{1/2} \sin E]\mathbf{Q}_s\} \cdot \mathbf{R} = -R^2 - r_c R \sinh \quad (8)$$

Under the assumption that $\mathbf{r}_m$ does not change by an appreciable amount in a period of about 12 hr, the approximate maximum upper time bound on the lunicentered satellite's time of observation, Eq. (8) reduces to a function of E and θ. The sidereal time θ enters through $\mathbf{R}$ by virtue of Eq. (1). The slight variation in $\mathbf{r}_m$ will be handled a little later in the analysis. To eliminate the time dependency of the $\mathbf{R}$ vector, consider the introduction of Kepler's equation, i.e.,

$$t = [(E - e_s \sin E)/n] + T_s \quad (9)$$

along with the relationship

$$\theta = \theta_0 + \dot{\theta}(t - t_0) \quad (10)$$

where t is the universal time, n is the mean motion $= k_m(\mu)^{1/2}\cdot a_s^{-3/2}$, k_m is the gravitational constant of the moon, μ is the sum of the masses of the lunicentered vehicle and moon, θ_0 is the epoch local sidereal time, t_0 is the universal time corresponding to θ_0, and $\dot{\theta}$ is the sidereal rate of change = const. Evidently, utilizing Eqs. (9) and (10), it is possible to define the local sidereal time θ in Eq. (1) as $\hat{\theta}$, where

$$\hat{\theta} = \theta_0 + (T_s - t_0)\dot{\theta} + (\dot{\theta}/n)(E - e_s \sin E) \quad (11)$$

so that as a function of E, Eqs. (1) become

$$\hat{X} = -G_1 \cos\phi \cos\hat{\theta} \qquad \hat{Y} = -G_1 \cos\phi \sin\hat{\theta}$$
$$\hat{Z} = -G_2 \sin\phi \quad (12)$$

These are the components of vector $\hat{\mathbf{R}}$, which is equivalent to $\mathbf{R}$, but is a function of E instead of θ. The controlling equation of the critical viewing time regime, i.e., Eq. (8), can therefore be expanded as

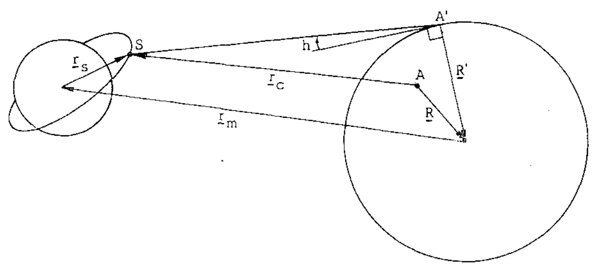

Fig. 2 Constraining geometry.

$$F \equiv \{x_m + a_s(\cos E - e_s)P_{xs} + a_s[(1 - e^2)^{1/2} \sin E]\, Q_{xs}\} \cos\phi \cos\hat{\theta} + \{y_m + a_s(\cos E - e_s)P_{ys} + a_s\,[(1 - e^2)^{1/2} \sin E]Q_{ys}\} \cos\phi \sin\hat{\theta} + \{z_m + a_s(\cos E - e_s)P_{zs} + a_s[(1 - e^2)^{1/2} \sin E]Q_{zs}\}\,(G_2/G_1) \sin\phi - (1/G_1)(r_c R \sinh + R^2) \quad (13)$$

The F function, Eq. (13), is the sought-for transcendental equation in the eccentric anomaly of the lunar satellite. Since $F \equiv \mathbf{r}_c \cdot \mathbf{R}$, and an obtuse angle exists between $\mathbf{r}_c$ and $\mathbf{R}$ when the satellite is visible, it follows that $F < 0$ is associated with the visibility condition. Hence, as F varies from negative to positive, the satellite and moon are setting with respect to the terrestrial station. A change of F from positive to negative, in opposite fashion, characterizes the satellite-moon system as rising with respect to the earth station. As the lunar satellite progresses through its period in eccentric anomaly, i.e., 0 to 2π, the F function will change sign every few periods.

The preceding equation defining F is fully rigorous with the exception of the Keplerian approximation dominating the motion of the lunar satellite. Keplerian motion for a lunar satellite is in effect a good approximation because the usual secular drift rates are not very prominent.

Solution of Eq. (13) is accomplished by a direct search technique, i.e., the condition $F = 0$ being the precise visibility condition. Hence, a starting value of E is chosen and Eq. (9) is employed to yield the universal time. An interpolation into a suitable ephemeris will yield $\mathbf{r}_m$, and Eq. (10), the local sidereal time, can be used to compute $\mathbf{R}$. Equation (2) immediately yields $\mathbf{r}_c$. A check is now made to see if $F = 0$; if so, a critical condition has been found; if $F \neq 0$, E is incremented and the procedure repeated. As soon as the critical eccentric anomalies have been determined, $\mathbf{r}_{mi}$ for $i = 1, 2$ can be recalculated, and a more correct value can be used in Eq. (13). In passing, it is well to note explicitly that when $h \neq 0$, Eq. (13) becomes even more functionally dependent on E since r_c must be obtained from $\mathbf{R}$, $\mathbf{r}_m$, and $\mathbf{r}_s$, i.e.,

$$r_c = (R^2 + r_m^2 + r_s^2 + 2\mathbf{R}\cdot\mathbf{r}_m + 2\mathbf{R}\cdot\mathbf{r}_s + 2\mathbf{r}_m\cdot\mathbf{r}_s)^{1/2}$$

It should be noticed that once the rise eccentric anomaly of the satellite-moon system has been found (or set eccentric anomaly), it is not necessary to continue the point-by-point critical viewing time regime check for the next x revolutions. In essence, since the period of the lunar satellite is known, a lower bound x can be found easily, where x is the number of revolutions from, for example, rise of the satellite-moon system to the revolution before set of the satellite-moon system. It is at this point that the checking procedure is reinitiated.

In closing, it should be evident that if $\mathbf{r}_s$ is taken to be zero, i.e., when the lunar satellite radius vector correction is neglected, Eq. (13) will yield the rise and set time[3] of the center of the moon with respect to a terrestrial station.

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Hugoniot Equations of State for Plastics: A Comparison

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THE Hugoniot equation of state is one of the constitutive relations describing the response of a material to hypervelocity impact. In recent years, impact at hypervelocities (in excess of 15,000 fps) has assumed a more important role because of the exposure of artificial satellites to impact of micrometeorites and because of the possible use as a concept of defensive and offensive missile systems. (Most hypervelocity impact experiments using the flying plate are extremely costly to conduct; therefore, a number of attempts have been made to correlate empirically and then to extrapolate the experimental data.) This paper compares these empirical equations of state for some plastics.

The term equations of state (sometimes called constitutive relations) means the relationships between the physical variables that are characteristic of the material involved. In gasdynamics, there is only one constitutive relation, the pressure-volume-temperature relation. In solid mechanics, the most general constitutive relations express the functional relationships among the stress components, strain components, stress-rate components, strain-rate components, and the specific internal energy. To arrive at a workable equation of state, several simplifying assumptions must be made.

If the stress state for which the equation of state is determined is assumed to be hydrostatic (i.e., uniform compressive stress), the appropriate equation of state is one in which the hydrostatic pressure replaces the stress components and the density (or compression ratio) replaces the strain components. Since the stress and strain components associated with any general state of stress can be broken into two components, dilatational (simple expansion or contraction) and distortional, the hydrostatic equation of state can be thought of as a part of the general stress-strain-temperature relations.

This assumption of hydrostatic condition is a reasonable one, since, in the initial phase of hypervelocity impact of two solids, the stresses are very much greater than the yield stress in shear so that at least for this time phase, a hydrostatic Hugoniot equation of state appears to be pertinent. Therefore, a hydrostatic Hugoniot equation of state will be referred to simply as the "equation of state" in this paper.

Most empirical equations of state are approached in the following manner. The conditions of conservation of mass and of momentum across a shock front (sometimes called the Rankine-Hugoniot relations) can be written as follows:

$$\rho/\rho_0 = U_s/(U_s - u_p) \qquad \text{(conservation of mass)}$$

$$P - P_0 = \rho_0 U_s u_p \qquad \text{(conservation of momentum)}$$

where ρ is the density, U_s is the shock velocity, u_p is the particle velocity, P is the pressure, and the subscript 0 denotes initial conditions. These relations assume that the initial velocity U_0 equals zero.

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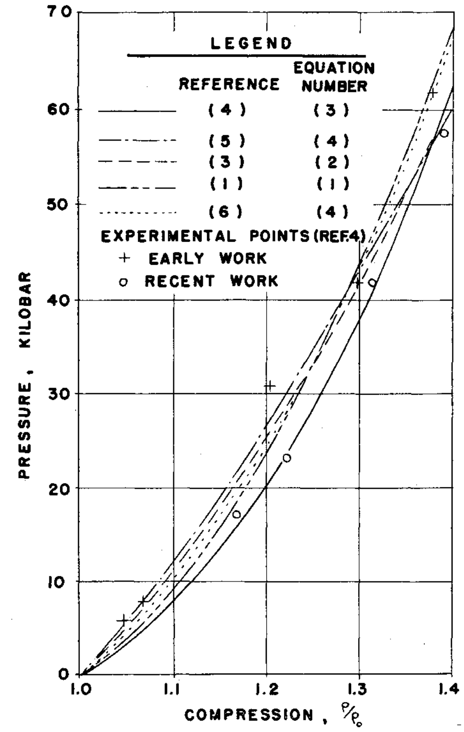

Fig. 1. Pressure-compression curves for polymethylmethacrylate including experimental data.

The relationship between the shock and particle velocities is generally assumed to be a linear one and has been found experimentally to be applicable to a variety of materials. This relationship is of the form

$$U_s = U_0 + \lambda u_p$$

where U_0 and λ are constants for a given material. The preceding expressions are then combined to develop an equation of state where the pressure $P = f(R)$ and $R = \rho/\rho_0$. In the following discussion, the constants are chosen to express P in kilobars.

Equations of State

In recent years, a number of seemingly different, empirical equations of state have been advanced. These equations are presented in chronological order with respect to date of publication. In 1958, Buchanan, James, and Teague[1] examined Perspex (a methyl methacrylate polymer) in compression and arrived at the following empirical expression:

$$P = 66.4(R - 1) + 262(R - 1)^2 \qquad (1)$$

The authors claim that, in 1956, Lawton and Skidmore[2] presented experimental evidence in support of their expression. In 1961, Blewett[3] examined both Lucite (a methyl methacrylate polymer) and polyethylene and found that his results fitted this somewhat simpler expression:

$$P = A(R^\gamma - 1) + 10^{-3} \qquad (2)$$

where for Lucite, $A = 30.1$, $\gamma = 3.33$, and for polyethylene, $A = 2.30$ and $\gamma = 9.20$.

In 1962, three new empirical expressions for the equations of state were published. The first was presented by Wagner, Waldorf, and Louie[4] in the form

$$P = BR(R - 1)/(C - R)^2 \qquad (3)$$

The authors investigated numerous plastics and determined